AIAA Journal
Vol. 45, No. 7, July 2007

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Period-Doubling Phenomenon Observed in the Dynamic Stall Vortex Patterns

Sunetra Sarkar* and Kartik Venkatraman
Indian Institute of Science, Bangalore, India 560 012

DOI: 10.2514/1.29656

I. Introduction

DYNAMIC stall is a nonlinear unsteady aerodynamic phenomenon resulting in stall delay during the time dependent movement of an airfoil at angles of attack higher than its static stall angle. This is observed during helicopter forward flights, rapid pitching of fighter aircraft, or in turbine rotor blades, and so on. The most important features of the flowfield are the leading and trailing edge vortex structures. Their growth, evolution, subsequent shedding into the outer field, and their mutual interactions influence aerodynamic loads significantly during the dynamic stall process. Various past and recent studies have been dedicated to visualizing the vorticity flowfield as system parameters are varied, experimentally and numerically. One of the earlier studies on dynamic stall was reported by McCroskey et al. [1]. The influence of various airfoil profiles and leading edge geometries was investigated. Experimental investigations on rapidly pitching airfoils was reported by Walker et al. [2]. Numerical simulations for a rapidly pitching airfoil with a compressible flow model were presented by Visbal and Shang [3] and Visbal [4]. Numerical simulations for a sinusoidally pitching airfoil were taken up by Tuncer et al. [5]. The effect of reduced frequency, albeit in a low range of variation, was highlighted. Ohmi et al. [6] presented experimental as well as numerical simulation results for a sinusoidally oscillating airfoil. Among other parameters like the Reynolds number, mean angle of attack, and elastic axis, reduced frequency was found to be the most significant one. Akbari and Price [7] have also investigated a similar parametric variation in a sinusoidally oscillating airfoil, though at lower range of reduced frequencies.

In most of the above and in other dynamic stall literature, the influence of reduced frequency ($k = \omega c/V_\infty$) has not been investigated beyond 1.0. However, for reduced frequencies just beyond that, we observe period-doubling behavior in the vortex patterns. This is an important qualitative change for the vortex patterns which otherwise follow the oscillating body's periodic motion. In this note, we present this period-doubling phenomenon of the vorticity patterns. Such qualitative changes are the signature of a nonlinear dynamical system. To the best of our knowledge, this was not reported before.

A Lagrangian discrete vortex particle technique with a random diffusion model has been used to simulate the unsteady flowfield for an incompressible, viscous flow model [8–10]. This method uses a vorticity representation of the flowfield and therefore is especially suitable for representing dynamic stall. System equations of motion for the incompressible, viscous flow are expressed in terms of the vorticity.

II. Results

We do not present any validation results for the numerical technique in this short note. However, the technique is well established and has been widely used. The random walk discrete vortex method originally proposed by Chorin [8] has been later adopted in other discrete vortex studies [11]. Subsequently, the technique has been validated and implemented for various applications of unsteady, separated, incompressible flowfields [12–14]. A detailed description of the method, its implementation strategies, and some validation results are discussed in [15].

The following pitching motion is simulated:

$\alpha(t) = 15 - 10\cos(\omega t)$, where $\alpha(t)$ is the time varying angle of attack and ω is the frequency of oscillation. The Reynolds number is 10^4, and the pitching axis is located at the quarter-chord. The location of the pitching axis is expressed as a fraction of the chord, and by convention is negative if it is located fore of the midchord position. We consider reduced frequencies $k = \omega c/V_\infty$ from 0.5 up to as high as 4.0. The cycle starts at $\alpha(0) = 5$ deg, nose up. The first half-cycle is the "upstroke" cycle as the trailing edge moves downward and angle of attack increases up to 25 deg. Up to $k = 1.0$, the vortex patterns follow the periodic motion of the airfoil. That is, they also repeat themselves with the same frequency as that of the body's oscillation. However, beyond that, a period-doubling phenomenon in the vorticity evolution cycle is observed. As the reduced frequency is increased further, the same trend continues. However, when $k = 3.0$, the period one trend reverts back once again and continues to be so as k is increased further and until $k = 4.0$, we see no further period doubling or other qualitative changes in the vortex patterns.

We present the vorticity patterns in these three different ranges, for $k = 1.0$, $k = 2.0$, and $k = 4.0$. We run quite a few cycles of simulation and so the initial transient effect goes away and the vortex pattern converges.

The reduced frequency case 1.0 is presented in Fig. 1. This can be compared with the results of Akbari and Price [7]. The vortex patterns at this reduced frequency match quite well with their results. Figure 1 shows snapshots from the upstroke and downstroke halves of the third cycle. During the early stages of the upstroke, the flow is separated and significantly vortical at the rear part of the body and near wake. These are the effects of the vorticity created and shed during the previous cycle. As the upstroke continues, the vorticity present on the rear part of the body is convected downstream and the reattachment process starts from the leading edge. This is seen in Fig. 1b, at an angle of attack of $\alpha = 16$ deg (upstroke). As the angle of attack increases, a trace of leading edge vortex is seen at $\alpha = 22$ deg. This vortex continues to grow, as seen in Fig. 1d during the rest of the upstroke. During the first part of the downstroke, presented in Figs. 1e and 1f, the leading edge vortex continues to grow and covers most of the upper surface by $\alpha = 16$ deg

Received 9 January 2007; revision received 25 March 2007; accepted for publication 26 March 2007.

*Research Student, Department of Aerospace Engineering; currently Postdoctoral Researcher, Technical University of Delft, Delft, The Netherlands; S.Sarkar@tudelft.nl; sunetra.sarkar@gmail.com.
†Associate Professor, Department of Aerospace Engineering.

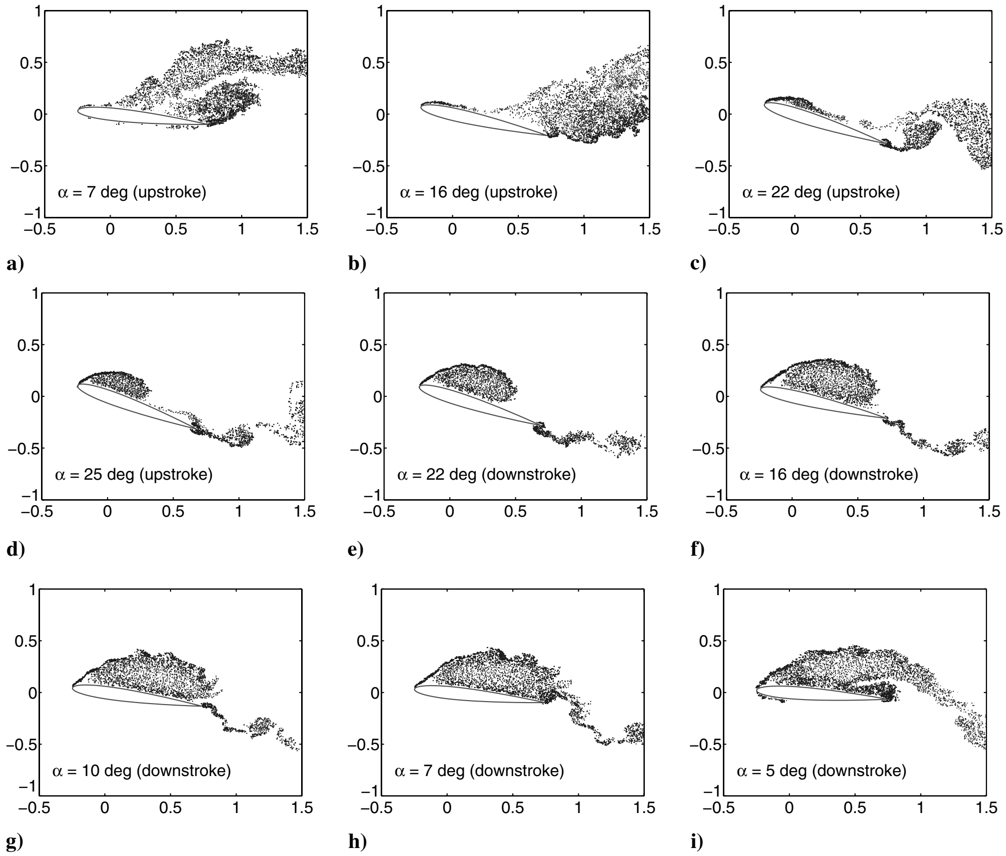

Fig. 1 Unsteady flow past airfoil pitching sinusoidally. $\alpha(t) = 15 - 10\cos(\omega t)$ (cycle 3); $k = 1.0$; $Re = 10^4$; $a = -\frac{1}{4}$.

(downstroke) as shown in Fig. 1f. Further on, it begins to separate very gradually from the body surface at about $\alpha = 10$ deg (downstroke), and a trace of a growing anticlockwise trailing edge vortex is also seen at this stage. During the rest of the downstroke, the trailing edge vortex grows and the shed leading edge vortex continues moving away from the body. This trailing edge vortex will grow further and will be shed later, as was already seen in the early upstroke snapshots. The load coefficient time histories are also presented in Figs. 2a and 2b. They compare well with other works published before, for example, by Akbari and Price [7].

Next, we present simulation results for higher reduced frequencies. It should be noted that as the frequency goes up, the effect of transient oscillation is more prominent. One needs to simulate for longer times or more number cycles to achieve convergence. The vortex patterns are described in the following. As the reduced frequency becomes 1.5, we observe the vortex patterns in the flowfield do not follow the time period of the airfoil movement any more. Instead, the vortex behavior takes two periods to repeat itself. We call this a period-doubling phenomena in the vortex patterns. This behavior is continued as the reduced frequency is increased even further to $k = 2.0$ and $k = 2.5$. We present the vortex patterns to highlight the period-doubling phenomenon for the $k = 2.0$ case. Results for the fifth and sixth cycles are presented with the help of a few vorticity snapshots. The simulation is carried through a total of 10 cycles. By this stage the total number of vortices accumulated in the entire computational domain becomes quite large and in turn significantly slows down the time marching process. However, one could use a particle reduction scheme [16,17] (though not included in the present algorithm) in the present algorithm to reduce the number of discrete particles. The vorticity snapshots for $k = 2.0$ are presented in Figs. 3 and 4 for the upstroke and downstroke halves of two consecutive cycles. Results are shown for the two consecutive cycles 5 and 6 and also the beginning of cycle 7. At the beginning of the upstroke of cycle 5, the near field is vortical and there is a sign of growing trailing edge vortex which separates from the body at around $\alpha = 16$ deg (upstroke). Toward the end of the upstroke, a leading edge vortex starts to grow and continues to do so throughout the downstroke. In the next cycle, this leading edge

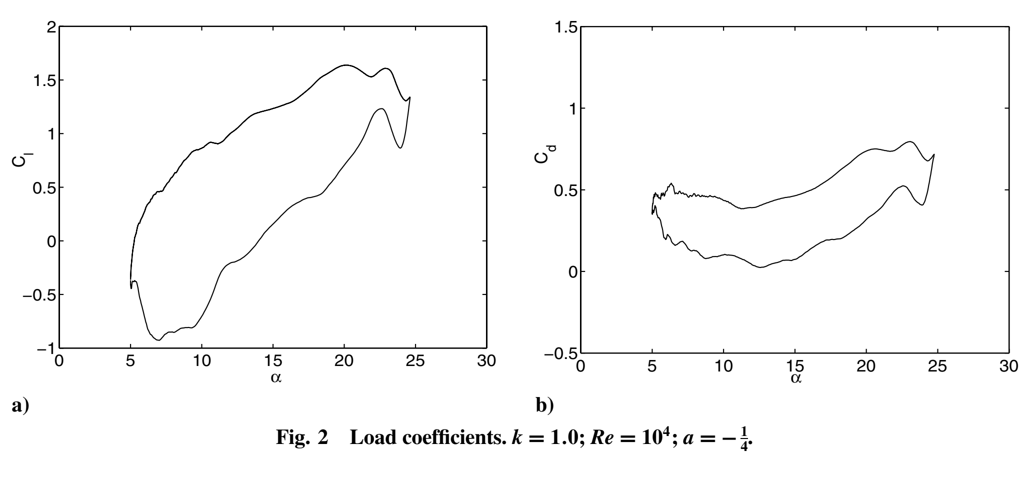

Fig. 2 Load coefficients. $k = 1.0$; $Re = 10^4$; $a = -\frac{1}{4}$.

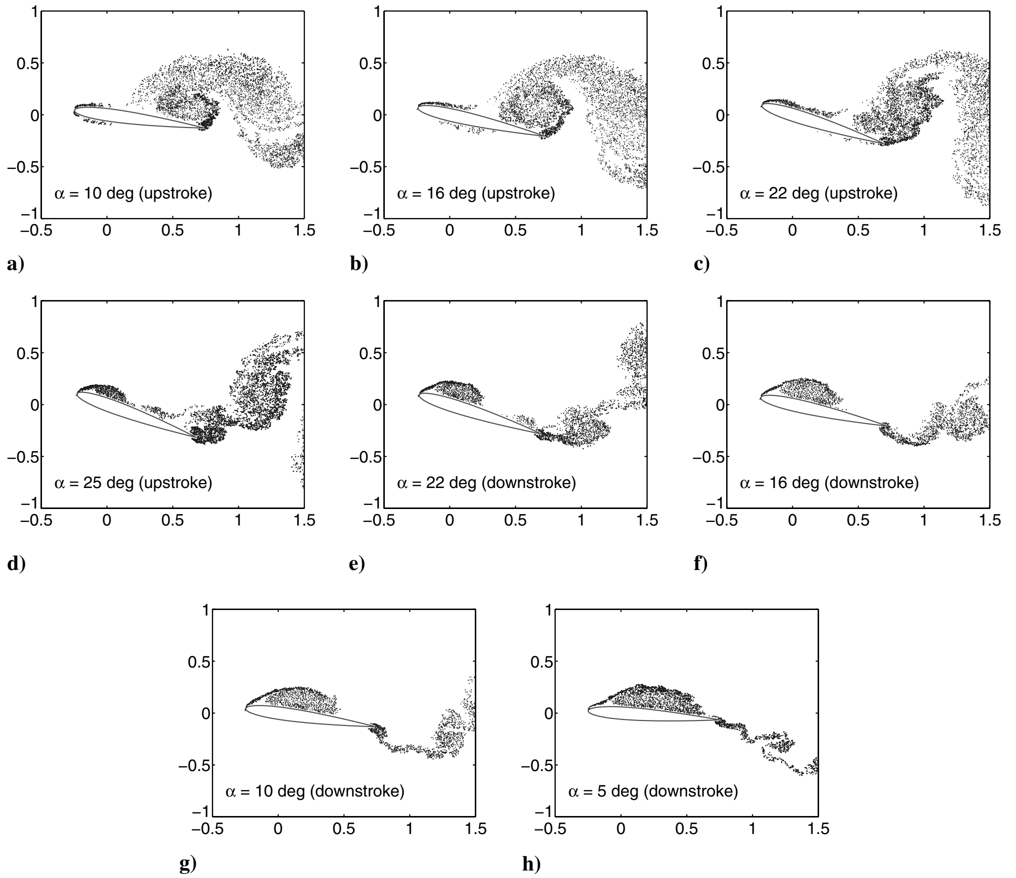

Fig. 3 Unsteady flow past airfoil pitching sinusoidally. $\alpha(t) = 15 - 10\cos(\omega t)$ (cycle 5); $k = 2.0$; $Re = 10^4$; $a = -\frac{1}{4}$.

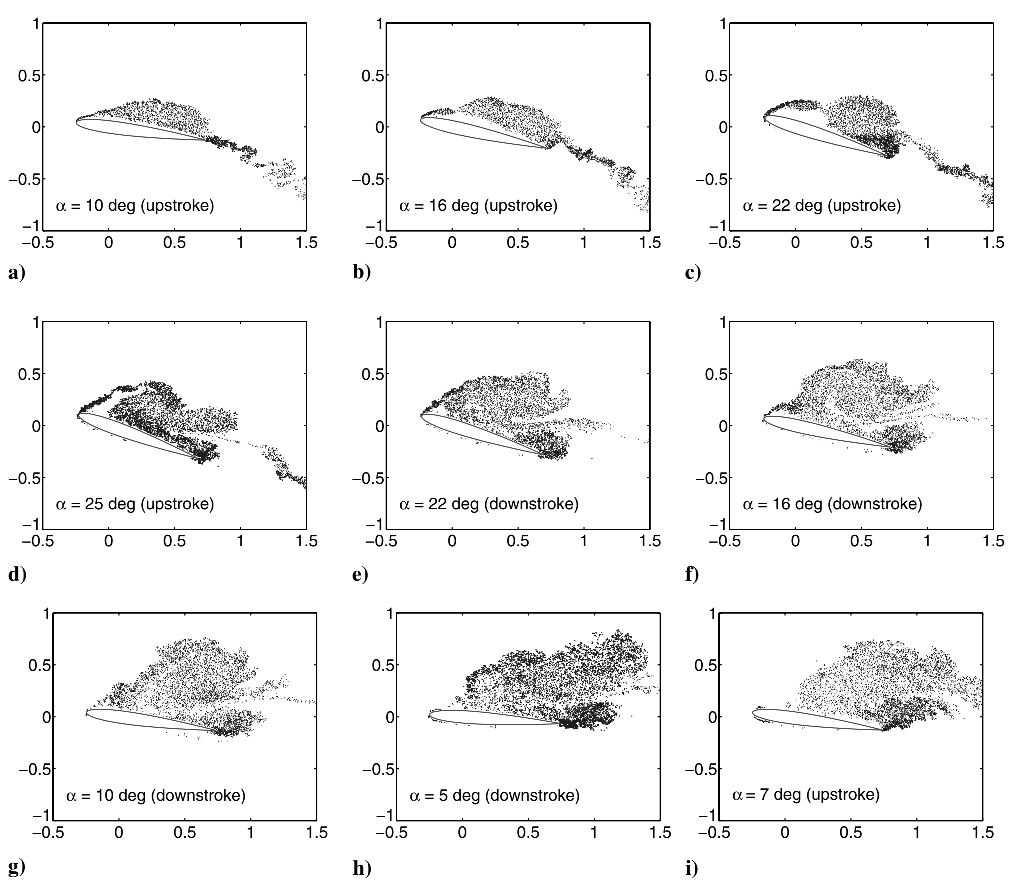

Fig. 4 Unsteady flow past airfoil pitching sinusoidally. $\alpha(t) = 15 - 10\cos(\omega t)$ (cycles 6 and 7); $k = 2.0$; $Re = 10^4$; $a = -\frac{1}{4}$.

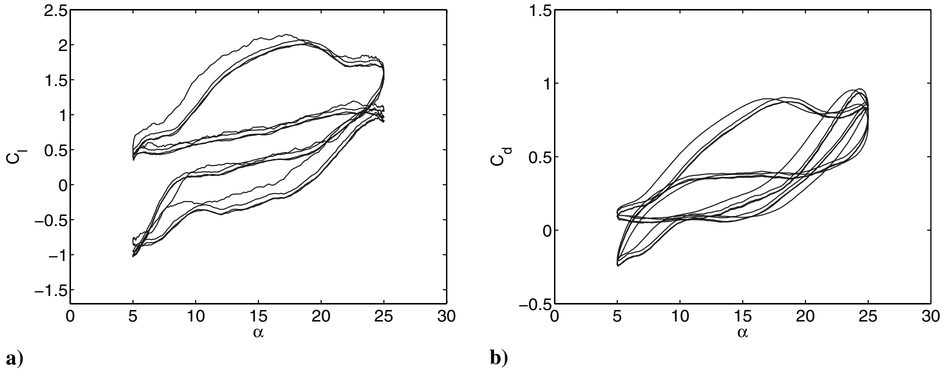

Fig. 5 Load coefficients. $k = 2.0$; $Re = 10^4$; $a = -\frac{1}{4}$.

vortex structure, which filled up most of the upper surface by the previous downstroke, begins to separate itself from the structure. At the same time a flow reversal starts from the trailing edge, which helps the leading edge vortex structure in separating (Figs. 4c and 4d). During the downstroke, the flow near the body remains vortical because of the presence of the just shed leading edge vortex structure. During this time, a trailing edge vortex structure also develops that continues to grow and finally gets shed in the next cycle. Thus, at the end of cycle 6, the vortex patterns resemble that with the start of cycle 5. The load coefficients time histories are presented in Figs. 5a and 5b. There are two loops indicating a period doubling. We plot the time histories for six consecutive cycles up to the tenth cycle to highlight the convergence of the loading. As k is increased to 3.0, we see the vortex patterns once again could follow the body's oscillation period. This is continued when $k = 4.0$ as well.

We present the vortex patterns for the $k = 4.0$ case in Fig. 6 for cycle 6 and also the start of the next cycle 7. The simulation is continued until the tenth cycle. The upstroke half-cycle starts with a leading edge vortex moving away from the body upper surface. A flow reattachment starts from the leading edge around $\alpha = 16$ deg (upstroke). Toward the end of the upstroke, a leading edge vortex structure is clearly seen to grow, as is seen in Fig. 6d. During the downstroke half-cycle, this leading edge vortex continues to grow. By almost the end of the downstroke, (see Fig. 6h), this vortex structure covers almost half of the upper surface. Meanwhile a small trailing edge vortex was also growing during the downstroke that gets shed around $\alpha = 10$ deg (downstroke). At the next upstroke, both the leading edge and the trailing edge vortex structures are seen to be moving away from the body. It is clear from the vortex snapshots that they come back to the same pattern after one period of oscillation. The loading time histories are shown in Figs. 7a and 7b for the last four cycles of simulation. The load time histories are shown to have converged to a single loop pattern indicating a period-1 behavior.

III. Summary and Conclusions

We present a period-doubling phenomenon observed during the dynamic stall process of a symmetric airfoil at high reduced frequencies. Such behavior is a signature of a nonlinear dynamical

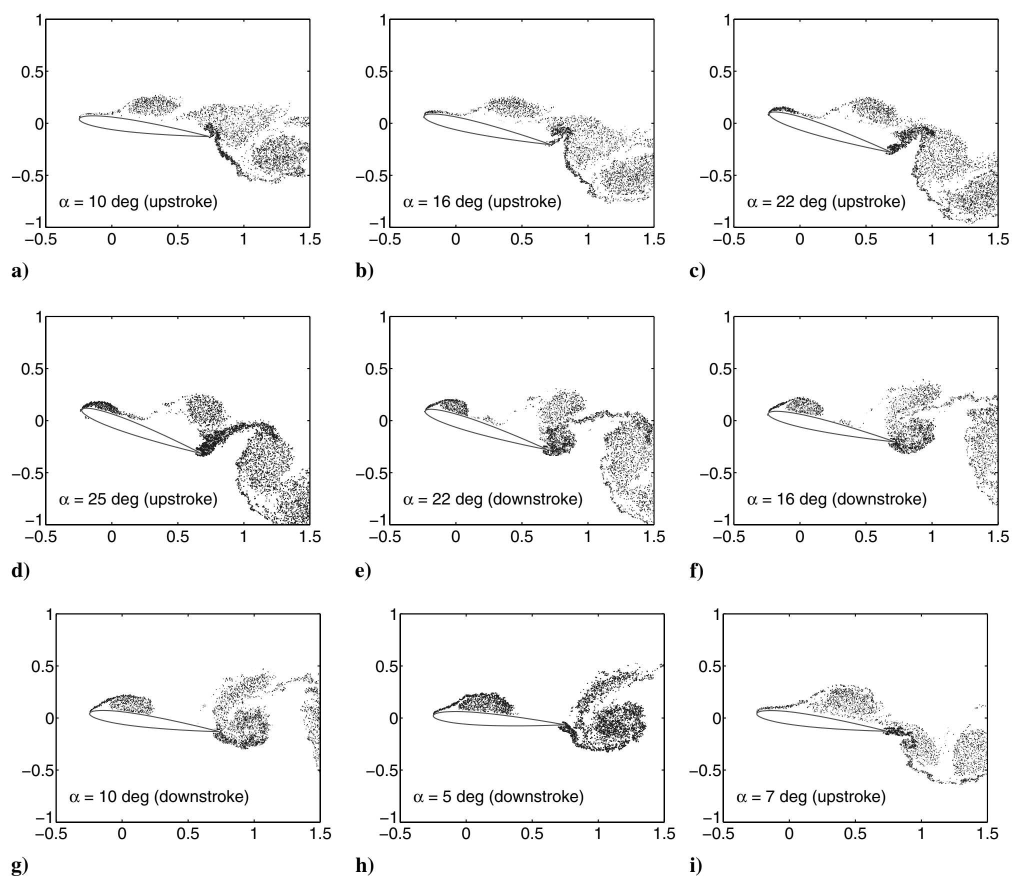

Fig. 6 Unsteady flow past airfoil pitching sinusoidally. $\alpha(t) = 15 - 10\cos(\omega t)$ (cycles 6 and 7); $k = 4.0$; $Re = 10^4$; $a = -\frac{1}{4}$.

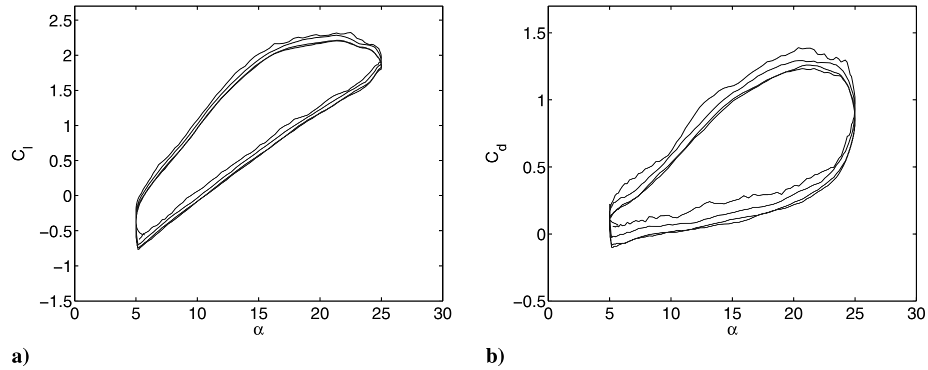

Fig. 7 Load coefficients. $k = 4.0$; $Re = 10^4$; $a = -\frac{1}{4}$.

system. The dynamic stall process involves complex flow physics and is highly nonlinear. Here the main players are the leading and trailing edge vortex structures and their evolution, movement, and mutual interactions in the flowfield. This mainly constitutes the near-field vortex patterns and controls aerodynamic loads. Reduced frequency is indeed an important parameter to influence the process, as was also reported in earlier works. However, previous studies have focused on a relatively low range of reduced frequencies where no period doubling was seen. In this work, we consider reduced frequencies in a moderate-to-high range. To the best of our knowledge, such period-doubling behavior was not reported before in the dynamic stall literature. As the frequency of oscillation goes up, as a result of the faster movement of the body in the flow, it becomes difficult for vortex structures to keep up with the body motion. This reflects on the higher periodic behavior of the vortex structures. However, it is interesting to note that at $k = 4$, the vortex structure again comes back to its original single periodic behavior. Thus we have observed a small window of double periodic response obtained through what is known as a flip bifurcation. It is also to be noted that the reduced frequency interval used here is common in dynamic stall analysis but may not be small enough for a traditional bifurcation analysis of nonlinear dynamics. It might be worthwhile to investigate at smaller intervals in case other qualitatively different solutions, for example quasiperiodic response, are present. This would then strengthen the argument that indeed the very fast movement of the body hinders the vortex structures to follow it up with the same time period. However, this would require a much larger number of cycles of simulation to identify the incommensurate frequencies of a quasiperiodic attractor. In this context we would also like to look at even higher reduced frequencies as a future course of study. It is also to be noted that in 3-D, the spanwise flow through the vortex cores can help stabilize the vortex core structures. As a result, this interesting period-doubling behavior may disappear. In the future, it would also be interesting to compare the vortex structures observed at high reduced frequencies with realistic 3-D mechanisms.

References

[1] McCroskey, W., Carr, L., and McAlister, K., "Dynamic Stall Experiments on Oscillating Airfoils," *AIAA Journal*, Vol. 14, No. 1, 1976, pp. 57–63.

[2] Walker, J., Helin, H., and Stricklan, J., "Experimental Investigation of an Airfoil Undergoing Large-Amplitude Pitching Motions," *AIAA Journal*, Vol. 23, No. 8, 1985, pp. 1141–1142.

[3] Visbal, M., and Shang, J., "Investigation of the Flow Structure Around a Rapidly Pitching Airfoil," *AIAA Journal*, Vol. 27, No. 8, 1989, pp. 1044–1051.

[4] Visbal, M., "Dynamic Stall of a Constant Rate Pitching Airfoil," *Journal of Aircraft*, Vol. 27, No. 5, 1990, pp. 400–407.

[5] Tuncer, I., Wu, J., and Wang, C., "Theoretical and Numerical Studies of Oscillating Airfoils," *AIAA Journal*, Vol. 28, No. 9, 1990, pp. 1615–1624.

[6] Ohmi, K., Coutanceau, M., Daube, O., and Loc, T., "Further Experiments on Vortex Formation Around an Oscillating and Translating Airfoil at Large Incidence," *Journal of Fluid Mechanics*, Vol. 225, April 1991, pp. 607–630.

[7] Akbari, M., and Price, S., "Simulation of Dynamic Stall for a NACA 0012 Airfoil Using a Vortex Method," *Journal of Fluids and Structures*, Vol. 17, No. 6, 2003, pp. 855–874.

[8] Chorin, A., "Numerical Study of Slightly Viscous Flow," *Journal of Fluid Mechanics*, Vol. 57, No. 4, 1973, pp. 785–796.

[9] Lin, H., Vezza, M., and Galbraith, R. McD., "Discrete Vortex Method for Simulating Unsteady Flow Around Pitching Aerofoils," *AIAA Journal*, Vol. 35, No. 3, 1997, pp. 494–499.

[10] Sarkar, S., and Venkatraman, K., "Numerical Simulation of Incompressible Viscous Flow past a Heaving Airfoil," *International Journal for Numerical Methods in Fluids*, Vol. 51, No. 1, 2006, pp. 1–29.

[11] Sarpkaya, T., "Computational Methods with Vortices: The 1988 Freeman Scholar Lecture," *Journal of Fluid Engineering*, Vol. 111, March 1989, pp. 5–52.

[12] Borthwick, A., and Barber, R., "Numerical Simulation of Jet Forced Flow in a Circular Reservoir Using Discrete and Random Vortex Methods," *International Journal for Numerical Methods in Fluids*, Vol. 14, No. 12, 1992, pp. 1453–1472.

[13] Ghoniem, A., and Ng, K., "Numerical Study of the Dynamics of a Forced Shear Layer," *Physics of Fluids*, Vol. 30, No. 3, 1987, pp. 706–721.

[14] Murgai, N., and Haan, F. J., "Gust Response of Separated Shear Layers Using Discrete Vortex Methods," AIAA Paper 2006-1298, 2006.

[15] Sarkar, S., "Incompressible, Viscous Flow Past An Oscillating Airfoil," Ph.D. Thesis, Indian Institute of Science, Bangalore, 2005.

[16] Van Dommelen, L., and Rundensteiner, E., "Fast, Adaptive Summation of Point Forces in the Two-Dimensional Poisson Equation," *Journal of Computational Physics*, Vol. 83, No. 1, 1989, pp. 126–147.

[17] Greengard, L., and Rokhlin, V., "Fast Algorithms for Particle Simulations," *Journal of Computational Physics*, Vol. 73, No. 2, 1987, pp. 325–348.

D. Gaitonde
Associate Editor